

Discussion Problems 7

Problem One: Undecidable Problems

Prove that $ENTERS = \{ \langle M, w, q \rangle \mid q \text{ is a state in } M \text{ and } M \text{ enters } q \text{ when run on } w \}$ is undecidable.

Problem Two: Infinity is Strange

Prove that $INFINITE = \{ \langle M \rangle \mid \mathcal{L}(M) \text{ is infinite} \}$ is not co-RE.

Problem Three: Sets and Subsets

The language A_{ALL} is defined as $A_{ALL} = \{ \langle M \rangle \mid \mathcal{L}(M) = \Sigma^* \}$. $A_{ALL} \notin \mathbf{RE}$ and $A_{ALL} \notin \mathbf{co-RE}$ (you'll prove this in Problem Set 8).

Prove that the language $SUBSET_{TM} = \{ \langle M_1, M_2 \rangle \mid M_1 \text{ and } M_2 \text{ are TMs, and } \mathcal{L}(M_1) \subseteq \mathcal{L}(M_2) \}$ is neither **RE** nor **co-RE** by reducing A_{ALL} to $SUBSET_{TM}$.